$$\hat{\sigma}_{11}+\hat{\sigma}_{22}+\hat{\sigma}_{33}=2\mu\,\mathrm{div}\,v+3\bar{\mu}\,\mathrm{div}\,v=3\,\mathrm{div}\,v(\bar{\mu}+\frac{2}{3}\mu)$$

$$\mathrm{Sp}\,\hat{\sigma}$$

$$p_{11}+p_{22}+p_{33}=-3p+\hat{\sigma}_{11}+\hat{\sigma}_{22}+\hat{\sigma}_{33}=\mathrm{Sp}\,\Pi$$

$$\mathrm{Sp}\,\Pi=-3p+3\,\mathrm{div}\,v\cdot\zeta\ ;\quad \zeta=\bar{\mu}+\frac{2}{3}\mu$$

$$\Pi=\begin{pmatrix} -p+\underbrace{2\mu\frac{\partial v_1}{\partial x_1}-\frac{2}{3}\mu\,\mathrm{div}\,v}_{\tau_{11}}+\zeta\,\mathrm{div}\,v & \mu\left(\frac{\partial v_1}{\partial x_2}+\frac{\partial v_2}{\partial x_1}\right) \\ & -p+\underbrace{2\mu\frac{\partial v_2}{\partial x_2}-\frac{2}{3}\mu\,\mathrm{div}\,v}_{\tau_{22}}+\zeta\,\mathrm{div}\,v \end{pmatrix}$$

$$-3p+\underbrace{2\mu\left(\frac{\partial v_1}{\partial x_1}+\frac{\partial v_2}{\partial x_2}+\frac{\partial v_3}{\partial x_3}\right)-3\cdot\frac{2}{3}\mu\,\mathrm{div}\,v}_{0=\mathrm{Sp}\,\tau}+3\zeta\,\mathrm{div}\,v=\mathrm{Sp}\,\Pi$$

$$\Pi=-pI+\zeta\,\mathrm{div}\,v\cdot I+\tau$$

$$\hat{\sigma}_{ij}=\begin{cases}\mu\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right), & i\neq j\\ \mu\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right)+\bar{\mu}\,\mathrm{div}\,v, & i=j\end{cases}\Rightarrow$$

$$\Rightarrow\ \hat{\sigma}_{ij}=\mu\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right)+\bar{\mu}\,\mathrm{div}\,v\cdot\delta_{ij}$$

$$\tau_{ij}=\mu\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right)-\frac{2}{3}\mu\,\mathrm{div}\,v\cdot\delta_{ij}$$

$$\Pi=-pI+\tau+\zeta\,\mathrm{div}\,v\cdot I$$

$-pI$ — чистое давление; τ — деф-я без изм. объема; $\zeta\,\mathrm{div}\,v\cdot I$ — отвечает за изм-е объема

$$\frac{d}{dt}(\rho v)=\mathrm{div}\,\Pi;\quad \frac{d}{dt}(\rho v)=\mathrm{div}\left(-pI+\tau+\zeta\,\mathrm{div}\,v\,I\right)$$

$$\frac{dv_i}{dt}=\frac{\partial v_i}{\partial t}+\sum_k v_k\frac{\partial v_i}{\partial x_k}$$ — с учётом ур-я нераз.

$$\Rightarrow\ \rho\frac{dv_i}{dt}=\sum_{k=1}^{3}\frac{\partial}{\partial x_k}\left(-p\,\delta_{ik}+\tau_{ik}+\zeta\,\mathrm{div}\,v\,\delta_{ik}\right)=$$

$$=\sum_{k=1}^{3}\frac{\partial}{\partial x_k}\left(-p\,\delta_{ik}\,\mu\left(\frac{\partial v_i}{\partial x_k}+\frac{\partial v_k}{\partial x_i}\right)-\frac{2}{3}\mu\,\mathrm{div}\,v\,\delta_{ik}+\zeta\,\mathrm{div}\,v\,\delta_{ik}\right)$$

$$\rho\frac{dv_i}{dt}+\frac{\partial}{\partial x_i}p-\frac{\partial}{\partial x_i}(\zeta\,\mathrm{div}\,v)=\sum_{k=1}^{3}\frac{\partial}{\partial x_k}\left(\mu\left(\left(\frac{\partial v_i}{\partial x_k}+\frac{\partial v_k}{\partial x_i}\right)-\frac{2}{3}\mathrm{div}\,v\cdot\delta_{ik}\right)\right)$$

Ур-е Навье Стокса (покомпонентно)

① $\mu = const$; $\zeta = const$

$$\sum_{k=1}^{3} \frac{\partial}{\partial x_k}\left(\frac{\partial v_i}{\partial x_k}\right) = \sum_{k=1}^{3} \frac{\partial^2 v_i}{\partial x_k^2} = \Delta v_i;$$

$$\sum_{k=1}^{3} \frac{\partial}{\partial x_k} \cdot \frac{\partial v_k}{\partial x_i} = \frac{\partial}{\partial x_i} \sum_{k=1}^{3} \frac{\partial v_k}{\partial x_k} = \frac{\partial}{\partial x_i} div\, v;$$

$$\sum_{k=1}^{3} \frac{\partial}{\partial x_k} div\, v \cdot \delta_{ik} = \frac{\partial}{\partial x_i} div\, v$$

$$\rho \frac{\partial v_i}{\partial t} + \frac{\partial}{\partial x_i} p = \frac{\partial}{\partial x_i} \zeta\, div\, v + \mu \Delta v_i + \mu \frac{\partial}{\partial x_i} div\, v - \frac{2}{3}\mu \frac{\partial}{\partial x_i} div\, v;$$

$$\rho \frac{\partial v_i}{\partial t} + \frac{\partial}{\partial x_i} p = \mu \Delta v_i + \left(\zeta + \frac{1}{3}\mu\right) \frac{\partial}{\partial x_i} div\, v;$$

$$\zeta = \bar{\mu} + \frac{2}{3}\mu \Rightarrow \zeta + \frac{1}{3}\mu = \bar{\mu} + \mu;$$

$$\rho \frac{dv}{dt} + grad\, p = \mu \Delta v + \left(\zeta + \frac{1}{3}\mu\right) grad\, div\, v;$$

② $\rho = const$ (несжимаемая жидкость)

$$\frac{\partial \rho}{\partial t} + div(\rho v) = 0 \Rightarrow div = 0 \Rightarrow$$

$$\Rightarrow \rho \frac{dv}{dt} + grad\, p = \mu \Delta v$$